Region IV, Northeastern MA
Climate Science Award:

Noah Arbesfeld, Lexington High School, Lexington

"On the Structure of Lower Central Quotients of a Free Associative Algebra"

ABOUT ME:

Noah Arbesfeld was born in Massachusetts and grew up in Lexington with his mother, father, and two younger brothers. An avid math and science student, he is a co-captain of his school's Math and National Science Bowl teams. In addition to participating in math and science competitions, Noah enjoys competing with his school's Lincoln-Douglas debate team. Noah also loves travel, geography, French, saxophone, and Shakespeare. Ultimately, Noah hopes to continue studying math and physics in college.

MY PROJECT:

Non-commutative algebra is an important branch of mathematics, with many applications to nearly all areas of mathematics, as well as quantum physics and chemistry.

In general, commutativity means that one can change the order of two processes without affecting the end result. For example, the multiplication and addition of real numbers is commutative, but matrix multiplication is not. More generally, noncommutative algebra arises any time one considers symmetries of sufficiently complicated objects.

Noah's project sought to better understand the fine structure of noncommutative algebras. The project considered free algebras, in a sense the "simplest" that one can construct. Though free algebras are important as they surject ["map" or are true for] onto all other algebras, they are extremely difficult to analyze in a nice, linear manner. However, when one a considers a construction called the Lower Central Series, which "measures", step-by-step, the free algebra's failure to be commutative, one notices a striking family of symmetries. Using both computational and theoretical tools, Noah's project determined the exact structure of steps of this series in many new cases, and also illustrated how one step of this series can link to each other, giving a more complete picture of the complicated structure of free algebras.

Editor's Notes: What is commutivity?: The algebra of real numbers (elementary algebra) that we first encounter in school is commutative: the order of operations of multiplication and of addition doesn't make a difference (5*2*3 = 3*2*5, and 5+2+3 = 3+2+5). Such properties tell us when we can rearrange an equation without changing it, so that we can find a solution: when only "x" is on one side of the equation and its value is on the other.

What is a matrix? When there many factors operating at the same time, resulting in many, many equations that need to be simultaneously true, mathematicians, engineers, and other scientists use a matrix to keep track of all of the coefficients of the parameters of the multiple equations. The matrix is an array, where every row consists of the coefficients of all of the variables for a particular equation (e.g. 1x +2y = 7 would have a row: 1, 2, 7); the columns are the parameters of one variable in the different equations, i.e. the first column would list the value for "x" in the first equation, the value for "x" in the second equation, etc..

If there are non-linear components, i.e. x^2 or y^2, then you can multiply by a second matrix to describe the total set of equations and variables. Matrices have their own algebraic properties, which can allow one to simplify finding the solution. However there are also restrictions that are not present in elementary algebra: with a matrix the order in which you carry out the operation of multiplication is important.

This project is in the field of theoretical mathematics, where there are few numbers, only symbols: Noah is examining the basic structure of algebra itself. Although this may seem to be an exercise in pure logic, such work helps us understand what is allowed in working in practical applications (just think of all of those proofs in geometry!). We are increasingly aware of the need to look at systems in a holistic fashion; instead of simplifying, we need to include all of the interactions and variables in our calculations and modeling. This is especially important in the area of climate change, where every year we realize the importance of a factor that we had previously overlooked.

A better understanding of the basic properties of algebra will lead to better ways of working with such complex, interactive systems.

According to an article in the Lexington Minuteman, Noah first got started on the project after attending the Massachusetts Institute of Technology's Research Science Institute (RSI), a summer program for high school students. This was where he met his mentor, Pavel Etingof, a professor of mathematics at MIT, and David Jordan, a graduate student at MIT. Noah was a finalist in Intel Science Talent Search 2009, and ranked six among the final top ten (www.cogito.org/Articles/ArticleDetail.aspx?ContentID=17690).

The work on this project has been submitted for publication, with David Jordan as co-author. (http://arxiv.org/abs/0902.4899)